SINR Diagram with Interference Cancellation

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Wireless Radio Networks

- Wireless devices
- Same Frequency and Time slots
- Wireless channel
- No centralized control
Physical Models: Signal to interference & noise ratio

Station $s_i$ is heard at $p$ iff

$$SINR(s_i, p) \geq \beta$$

Received signal strength of station $s_i$

$$\psi_i = 1 \text{ for every } i$$

Interference

$$\sum_{j \neq i} \frac{\psi_j}{dist(s_j, p)^\alpha} + N$$

$$SINR(s_i, p) = \frac{\psi_i}{dist(s_i, p)^\alpha}$$
A map characterizing the reception zones of the network stations

Reception Zone of Station $s_i$

$$H(s_i) = \{s_i\} \cup \left\{p \in R^d - S \mid \text{SINR}(s_i, p) \geq \beta\right\}$$

Null Zone

$$H_{\phi} = \left\{p \in R^d - S \mid \text{SINR}(s_i, p) < \beta, \text{for every } s_i\right\}$$

Cell := Maximal connected component within a zone.
Unit Disk Graph (UDG) model Vs SINR model

UDG yields to “false positive” indication of reception
Unit Disk Graph (UDG) model
Vs
SINR model (cont.)

"false negative" indication
Successive Interference Cancelation (SIC)

Signal A

0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 0, 1, 1, ...

0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, ...

Signal B

Received Signal

Key Idea: If you can’t fight it, Cancel it!
Decode the Strongest Signal

Received Signal

Decode (Rounding)

Signal A

0,0,0,0,0,1,0,1,0,1,0,1,1,1,1,0,0,1,1,...
and subtract (cancel) it ...

Received Signal

Signal

Interference

Amplify

0,0,0,1,0,0,0,0,1,1,0,1,0,0,1,1 ...

...
All stations transmit with power 1

\((\psi_i = 1 \text{ for every } i)\)

**Theorem:**

- Every reception zone \(H(s_i)\) is **convex**.
- \(H(s_i) \subseteq \text{Vor}(i)\).

[Avin et al. PODC09]

[Avin, Emek, Kantor, Lotker, Peleg, Roditty, PODC 2009]
After $S_2$ cancellation, now $S_3$ is cancelled.

Reception zone of $S_1$ after all possible cancellations.

SINR Diagram with IC

SINR Diagram with IC

Decode $S_2$ and then $S_1$

Decode $S_2$ and then $S_3$

Decode $S_2$, then $S_3$ and finally $S_1$

Decode $S_3$, then $S_2$ and finally $S_1$

SINR Diagram with IC

SINR Diagram with IC

Decode $S_3$ and then $S_1$
SIC-SINR reception zones are not convex in $\mathbb{R}^d$.

Apriori exponential number of cancellation orderings.
Challenges

Topology
- Bounding #Cells (how many?)

Algorithms
- Map Drawing (which are they?)

Nice Properties of Zones
- Point Location Task (how to store them?)
Reception Region of Cancellation Ordering

- Ordering of $k \leq n$ stations

- The reception region of points that share the same order of cancellation

\[ \tilde{S}_i = (s_1^i, s_2^i, \ldots, s_k^i) \]

\[ H(s_1 | S - \{s_2\}) \cap H(s_2) = H(s_2, s_1) \]
Reception Region of Cancellation Ordering (CO): General case

\[ H(\tilde{S}_i) = H\left(s^i_k \mid S - \{s^i_1, \ldots, s^i_{k-1}\}\right) \cap H\left(s^i_1, \ldots, s^i_{k-1}\right) \]

- Convex !!
- Reception zone in Uniform network (without SIC)
- Convex by Induction
High-Order Voronoi Diagram

- Extension of the ordinary Voronoi diagram
- Cells are generated by more than one point in S.
- Every region consists of locations having the same closest points in S.

Order $k=2$

Some Voronoi polygons of order two are empty. For example, there is no (5,7) polygon.

Cell’s generators

M. Shamos and D. Hoey, FOCS '75
\[\vec{S}_i = (s^i_1, s^i_2, \ldots, s^i_k)\]

**Lemma**

[High Order and SIC-SINR]

\[H(\vec{S}_i) \subseteq \text{Vor}(\vec{S}_i)\]

**Lemma [High-Order cells]**

There are \(O(n^{2d})\) orderings \(\vec{S}_i\) such that \(\text{Vor}(\vec{S}_i) \neq \emptyset\).
The Topology of Reception Zone with SIC

The Reception zone of every network’s station is a collection of $O(n^{2d})$ shapes.

Each shape is:

- Convex
- Correspond to distinct cancellation ordering
  $$\tilde{S}_i = (s_1^i, s_2^i, \ldots, s_1^i) \subseteq S$$
- Fully contained in the high-order Voronoi cell
  $$\text{Vor}(\tilde{S}_i)$$
$CP(A)$ Compactness Parameter of network $A$

$CP(A) = \sqrt[\alpha]{\beta}$

Path-loss exponent

Reception Threshold (>1)
Theorem [Linear #Cells]
If $CP(A) > 5$ then $H_1^{SIC}$ is composed of $O(1)$ cells for any dimension $d$.

$O(n^{2d})$ non–empty high order Voronoi cells,

Only $O(1)$ of them are non–empty reception cells!
Separating hyperplane of station $s_i, sj \in S$

\[ HP(s_i, sj) = \{ p \in \mathbb{R}^d | \text{dist}(s_i, p) = \text{dist}(sj, p) \} \]

The set of $\binom{n}{2}$ hyperplanes of pairs in $S$ dissects $\mathbb{R}^d$ into connected regions

We call this dissection $Ar(S)$ the arrangement of $S$.

**Theorem [Arrangements, Edelsbrunner 1987]**

- $Ar(S)$ contains $O(n^{2d})$ cells.
- $Ar(S)$ can be constructed in $O(n^{2d})$ time and place.
Label each cell $f \in Ar(S)$ with ordered list of $S$ in increasing distance from $f$.

Observe:

- Stations ordering is required only once.
- Subsequent labels are computed in $O(1)$.
Construct the labelled arrangement $Ar(S)$.

Extract cancellation ordering $\hat{S}_i$ of non-empty cells.

Derive the Characteristic Polynomial of $H(\hat{S}_i)$ (Intersection of $|S_i|$ polynomials).

Draw the resulting polynomial.

Efficiency: $O(n^{2d} + 1)$ in time & place.
Challenges – What We Know

Topology

#Cells
General - $O(n^{2d})$
Compact network - $O(1)$

Nice Properties of Zones
Collection of convex cells
Each related to high-order Voronoi cell.

Algorithms

Map Drawing
Efficient construction of labeled arrangement $O(n^{2d+1})$

Point Location Task
Poly-time construction.
Logarithmic time per query
\( H(s_2, s_3, s_1) \) in green, and \( H(s_3, s_2, s_1) \) in pink. The initial value of \( \beta \) is 1. If you observe the initial configuration of the stations you will notice that you do not see a “blue reception zone”. Why?

(answer: because this zone represents \( H(s_3, s_1) \) and with the actual position of each station this zone cannot exist. Because when you are in \( H(s_3) \) you cannot “hear” \( s_1 \) until you cancel before \( s_2 \). That is why \( H(s_3, s_2, s_1) \) is not empty whereas \( H(s_3, s_1) \) is empty. Now, if you rotate \( s_2 \), 180 degree, what do you see? ...)

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### Graphical Interface

**Zoom**

**\( \beta \)**

**No**
Questions?

Thank You!